



## Assignment 9, Complexity Theory, WiSe 16/17

Markus Bläser, Anurag Pandey, Harry Zisopoulos  
<http://www-cc.cs.uni-saarland.de/course/56/>

---

Due: Jan 26, 2017

---

**Exercise 9.1** a) Let  $R$  be an NP relation. Let  $\oplus R$  be the language

$$\{x \mid \text{the number of } y \text{ with } R(x, y) = 1 \text{ is odd}\}.$$

Prove that if  $\#R$  is  $\#P$ -hard under parsimonious reductions, then  $L(R)$  is NP-hard under many-one reductions and  $\oplus R$  is  $\oplus P$ -hard under many-one reductions.

b) Prove that we can decide in polynomial time whether the permanent of a  $\{0, 1\}$ -matrix is odd.

**Exercise 9.2** Let  $\mathcal{C}$  be a complexity class. Show that  $\text{co-BP-}\mathcal{C} = \text{BP-co-}\mathcal{C}$

**Exercise 9.3** Show that the following problem is  $\exists\oplus P$  complete: Given a formula in 3-CNF in variables  $y_1, \dots, y_n$  and  $z_1, \dots, z_m$ , is there an assignment to  $y_1, \dots, y_n$  such that the resulting formula has an odd number of satisfying assignments to  $z_1, \dots, z_m$ ?

**Exercise 9.4** Prove that  $P^{\#P} = P^{PP}$ .