Assignment 8, Complexity Theory, WiSe 16/17

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Exercise 8.1 For an undirected multigraph G, let $M(G;x)$ be the *matching polynomial* of G, that is, the polynomial

$$M(G;x) = \sum_{\text{matching } M \subseteq E(G)} x^{|M|}.$$

Recall that $M \subseteq E(G)$ is a *matching* if, for all $e, e' \in M$ with $e \neq e'$, we have $e \cap e' = \emptyset$.

a) Prove that the function $G \mapsto M(G;0)$ can be computed in polynomial time.

b) Let $m_k(G) \in \mathbb{N}$ be such that $M(G;x) = \sum_{k=0}^{n} m_k(G)x^k$. Prove that $G \mapsto m_n(G)$ is #P-hard under polynomial-time many-one reductions.

c) For all fixed $a \in \mathbb{Q} \setminus \{0\}$, prove that $G \mapsto M(G;a)$ is #P-hard to compute under polynomial-time Turing reductions.
Hint: First construct from G a graph G_t such that $M(G;tx) = M(G_t;x)$ for $t \in \mathbb{N}$. Then use polynomial interpolation to compute the coefficients from sufficiently many evaluation points.

Exercise 8.2 a) Let $X = (x_{i,j})$ be an $n \times n$-matrix and let

$$f = \prod_{i=1}^{n} \sum_{j=1}^{n} x_{i,j} Y^{2^{j-1}}.$$

Prove that the coefficient of Y^{2^n-1} in f is $\mathrm{perm}(X)$.

b) Prove that the permanent has algebraic circuits of size $\mathrm{poly}(n) \cdot 2^n$.

c) Let BitSLP be the following problem: Given a variable-free circuit computing an integer N and an index i, return the ith bit of N. Prove that BitSLP is #P-hard under polynomial time Turing reductions.