Assignment 5, Complexity Theory, WiSe 16/17

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In this home work, we will prove a interesting variant of the Karp–Lipton theorem due to Meyer.

Definition 1 *a) Let $x, y \in \{0,1\}^*$. x is* wdq-smaller *than y, in terms $x <_{\text{wdq}} y$, iff $|x| < |y|$ or $|x| = |y|$ and x is lexicographically smaller than y.*

b) A language A is wdq-self-reducible *if there is a polynomial time deterministic oracle Turing machine M such that $A = L(M^A)$ and for each input x, every word that is presented to the oracle is wdq-smaller than x.*

The term *wdq* stand for *word decreasing queries.*

Exercise 5.1 Let A and B be wdq-self-reducible. Show the following: If $A = L(M^A)$ and $B = L(M^B)$ for some deterministic polynomial-time Turing machine M that queries the oracle only about strings that are wdq-smaller than the input, then $A = B$

Exercise 5.2 Show the following: If $A \in \Sigma_i^P/\text{poly}$ and A is wdq-self-reducible, then $(\Sigma_2^P)^A \subseteq \Sigma_{i+2}^P$. In particular, $A \in \Sigma_{i+2}^P$.

We define the *computation tableau* of a deterministic Turing machine M with running time t. W.l.o.g. we assume that M has only one tape and if M accepts, it ends with the empty tape and its head is in the same position as it started. (This blows up the running time only by a polynomial factor.) Given an input x, the computation tableau corresponding to x basically is a matrix with $t(|x|)$ columns and $t(|x|) + 1$ rows. Each row is a configuration c of M, that is, it is a word of length $t(n)$ from $(\{0, 1, \Box\} \cup \{0, 1, \Box\} \times Q)^*$. (Here Q is the set of states of M.) Exactly one symbol of c is of the form (σ, q). This marks the position of the head and q is the current state of M. The symbols from $\{0, 1, \Box\}$ describe the content of the tape. The computation tableau corresponding to x now consists of these configurations such that the first row is the starting configuration with x written on the tape and the configuration in row $i + 1$ is the unique configuration that can be reached from the configuration in the ith row by one step of M. Define

$$\mathtt{CT} = \{\langle i, j, u, M, x\rangle \mid u \text{ is the entry in row } i \text{ and column } j \text{ of the computation tableau of } M \text{ on input } x\}.$$

Exercise 5.3 $\mathtt{CT}$ is EXP-complete and wdq-self-reducible.

Use the results above to show the following variant of the Karp–Lipton theorem.

Exercise 5.4 (Meyer) If $\mathsf{EXP} \subseteq \mathsf{P}/\mathsf{poly}$, then $\mathsf{EXP} = \Sigma_2^P \cap \Pi_2^P$.