Assignment 2, Complexity Theory, WiSe 16/17

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Exercise 2.1 A partial function $f : \{1, \dots, n\} \to \{1, \dots, n\}$ can be represented as a directed graph in which every node has outdegree at most 1. (For each i, simply add the edge $(i, f(i))$ whenever $f(i)$ is defined.) Prove that the following language is in L:

$$\texttt{FCONN} = \{(G, s, t) \mid G \text{ is a partial function without a cycle having a path from } s \text{ to } t\}$$

Exercise 2.2 A language L is NC_1-reducible to L', if there is a logarithmic space constructible family of circuits for L that is poly(n) size bounded and logarithmic depth bounded. In addition to the usual gates, this family may use *oracle* gates, that is, gates computing χ_L (restricted to some input size). If an oracle gate has i inputs, it add i to the size and $\log i$ to the depth of a circuit.

Prove that FCONN is L-complete under NC_1-reductions.
(Hint: Make the configuration graph acyclic by layering it.)

Exercise 2.3 Prove the following:

a) Let $\texttt{PARITY} = \{x \in \{0,1\}^* | x \text{ has an odd number of 1s}\}$. Prove that $\texttt{PARITY} \in \mathsf{NC}_1$.

b) Construct logspace uniform AC_0 circuits that add two n-bit integers.
(Hint: Write an AC_0 expression for the ith carry.)

c) Construct logspace uniform NC_1 circuits that add n n-bit integers.
(Hint: Write the sum of three n-bit integers $x+y+z$ as the sum of two $(n+1)$-bit integers $u+v$ such that $x_i 2^i + y_i 2^i + z_i 2^i = u_i 2^i + v_{i+1} 2^{i+1}$, where x_i, y_i, z_i, u_i, and v_{i+1} are the corresponding bits. Do this in parallel and recurse.)

Exercise 2.4 The *circuit value problem* CVAL is the following problem: Given (the encoding of) a circuit C with n input gates and one output gate, and a string $x \in \{0,1\}^n$, the goal is to decide whether $C(x) = 1$. Prove that CVAL is P-complete under logarithmic-space many-one reductions.

Hint: We already proved that $\texttt{CVAL} \in \mathsf{P}$. To show that CVAL is P-hard, first prove Remark 4.6 in the lecture notes.