Assignment 1, Complexity Theory, WiSe 16/17

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Due: Nov 10, 2016

If not stated otherwise, every exercise is worth 4 points (regardless of its difficulty).

Exercise 1.1 Prove that logarithmic space computable functions are closed under composition, that is, if $f, g : \{0,1\}^* \to \{0,1\}^*$ are both computable by a deterministic logarithmic space bounded Turing machine, so is their composition $f \circ g$.

Exercise 1.2 Proof that the language $L = \{e \mid e \text{ is the encoding of a log-space bounded TM}\}$ is not decidable. (There is nothing particular about log-space bounded, it works for many properties.)

Exercise 1.3 (Translation) Complete the proof of the Immerman–Szelepcsényi Theorem by showing the following: Let f be space constructible such that $f(n) \geq \log(n)$. If

$$\mathsf{NSpace}(\log(n)) \subseteq \mathsf{co\text{-}NSpace}(\log(n)),$$

then,

$$\mathsf{NSpace}(f(n)) \subseteq \mathsf{co\text{-}NSpace}(f(n)).$$

(Hint: Mimic the proof of the time translation done in class. If the space bounds are sublinear, then we cannot explicitly pad with %s. We do this virtually using a counter counting the added %s.)

Exercise 1.4 A function $s : \mathbb{N} \to \mathbb{N}$ is called fully space constructible, if there is a deterministic Turing machine that on every input of length n uses *exactly* $s(n)$ cells on the work tape.

a) Prove that $g(n) := \lceil \log_2(f(n)+1) \rceil$ is fully space constructible, where $f(n)$ is the smallest integer not dividing n.

b) Prove that $g(n) = O(\log \log n)$.

c) Prove that every fully space constructible function in $o(\log n)$ has to equal some constant value infinitely often.