Assignment 5, Complexity Theory, WS 13/14

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Exercise 5.1 Let φ be a satisfiable formula in CNF. Consider the following algorithm U:

a) Set $i := 0$

b) Enumerate all encodings g of deterministic Turing machines of length $\leq i$.

c) Simulate the Turing machine given by g for $2^{i-|g|}$ steps.

d) If the Turing machine halts with output y, check whether y is a satisfying assignment for φ.

e) If yes, accept. Otherwise set $i := i + 1$ and goto step a).

Let M be a deterministic Turing machine that outputs a satisfying assignment for φ in time $t_M(\varphi)$. Let t_U be the running time of U. Prove that $t_U(\varphi) \leq c_M t_M(\varphi) + \text{poly}(|\varphi|)$ for some constant c_M which only depends on M.

Exercise 5.2 Assume you prove $\mathtt{SAT} \in \mathsf{DTime}(n^3)$ with a nonconstructive proof. Is it really nonconstructive?

Exercise 5.3 The dependence of c_M in the first exercise is exponential in the length of the encoding of M. Prove that if there is an algorithm $\hat{U}$ which has the same property as above but the constant c_M is polynomial in the length of the encoding of M, then $\mathsf{P} = \mathsf{NP}$.