Complexity Theory, WS 13/14: Solution Hints 10.

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Exercise 10.1 Given a directed acyclic graph G with n vertices, the problem of counting a number of paths from source s to sink t is $\#\mathsf{L}$-hard.

Let A be an adjacency matrix of G, and S be a zero matrix except one element (s,s). Then $((S+A)^n)_{s,t}$ is a total number of paths from source s to sink t. This can be computed in polynomial time, and actually poly-logarithmic parallel time.

Exercise 10.2 If we can count the number of satisfying assignments of a formular in DNF, then we can do the same for CNF.

Given CNF φ with n variables, then we can easily compute DNF representing $\neg\varphi$. Let k be a number of assignments where $\neg\varphi = 1$, then $2^n - k$ is a number of assignments where $\varphi = 1$, which is a desired number.

We should not expect to have a parsimonious reduction . Otherwise, we have a reduction f such that, given a CNF φ, we can compute a DNF $f(\varphi)$ where φ is satisfiable iff $f(\varphi)$ is satisfiable. But checking if a DNF is satisfiable can be done in polynomial time.

Exercise 10.3 Let $\#matching$ be that problem of counting a number of matching (perfect or not) in bartite graph. Note that $\#matching$ is $\#\mathsf{P}$-hard under Turing reduction.

To show that $\#2SAT$ is $\#\mathsf{P}$-hard, we show $\#matching \leq_{par} \#2SAT$. Let $e_1, e_2, \ldots, e_m$ be all the edges in a graph G. For each pair of adjacent edges (e_i, e_j) in G, we construct a clause $(\neg x_i \vee \neg x_j)$ in our 2-CNF φ, hence x_i, x_j can be not be "true" at the same time corresponding to the fact that e_i and e_j can not be in any matching at the same time. So each matching in G corresponds to a satisfying assignment in φ. Thus, we get a parsimonious reduction.

As $\#2SAT$ is trivially in $\#\mathsf{P}$, $\#2SAT$ $\#\mathsf{P}$-complete.