Assignment 9, Complexity Theory, WS 13/14

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A *straight line program* (or SLP for short), is a sequence of instructions that correspond to a sequential evaluation of an arithmetic circuit. Formally an SLP is a sequence of instructions $g_{-n}, \dots, g_{-1}, g_1, \dots, g_m$, where $g_{-n}, \dots, g_{-1} \in \mathbb{Z}$ and for $1 \le i \le m$, either $g_i \in \mathbb{Z}$, or $g_i = (*, L_i, R_i)$, with $* \in \{+, \times, -\}$, $L_i, R_i \in \{g_{-n}, \dots, g_{-1}, g_1, \dots, g_{i-1}\}$. Naturally for every SLP we can associate an integer $value(g_i)$ with every instruction g_i. The integer represented by the given SLP is the value of g_m. We define two computational problems on SLPs:

EquSLP Given an SLP P representing an integer N, decide if $N = 0$.

BitSLP Given an SLP representing an integer N, $n, i \in \mathbb{N}$, test if the ith bit in the n-bit binary representation of N is 1.

PosSLP Given an SLP computing an integer N, test if N is positive, i.e, $N > 0$.

Exercise 9.1 Show that *EquSLP* is polynomial time many-one equivalent to ACIT.

Exercise 9.2 Show that *BitSLP* is #P hard under polynomial time Turing reductions. (Hint: $\prod_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x^{2^{j-1}}$.)

Exercise 9.3 (Csanky's algorithm) The characteristic polynomial of a matrix A is defined as $c_A(X) = \det(X \cdot I - A)$ where I is the identity matrix. Let $c_A(X) = s_{A,0}X^n + s_{A,1}X^{n-1} + \cdots + s_{A,n}$.

a) Show that

$$s_{A,0} = 1$$
$$s_{A,k} = \frac{1}{k} \sum_{\kappa=1}^{k} (-1)^{\kappa-1} s_{k-\kappa} \operatorname{trace}(A^{\kappa}), \qquad 1 \le k \le n.$$

b) Show that $s_{A,n} = \det A$.

c) Show that there is a logarithmic space uniform family of Boolean circuits of polynomial size and polylogarithmic depth that computes the determinant of a matrix A. (Assume that A has dimension $n \times n$ and entries with $p(n)$ bits for some polynomial p.)