Assignment 7, Complexity Theory, WS 13/14

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This sheet contains some example exam questions. You may submit them to get some extra points.

Exercise 7.1 Show that if $\mathsf{NSpace}(\log n) \cap \{0\}^* = \mathsf{DSpace}(\log n) \cap \{0\}^*$ then $\mathsf{NSpace}(n) = \mathsf{DSpace}(n)$.
(Hint: some sort of translation)

Exercise 7.2 Which of the following statements are true, which are false? Give a short proof for each of your answers!

a) $\{x \in \{0,1\}^* \mid x = x^{\text{rev}}\} \in \mathsf{DSpace}(\log \log \log n)$.

b) For every complexity class C and $A \in \mathsf{C}$: If A is C-complete under polynomial time many one reductions, then $\mathsf{C} \subseteq \mathsf{P}^A$.

c) If $\mathsf{NC}_2 \subseteq \mathsf{L}$, then $\mathsf{DSpace}(s(n)) = \mathsf{NSpace}(s(n))$ for all space constructible $s \geq \log n$.

d) If $\mathsf{PH} = \mathsf{PSPACE}$, then PH collapses.

e) There is an oracle B such that $\mathsf{PH}^B = \mathsf{PSPACE}^B$.

Exercise 7.3 `MaxSAT` is the following search problem: Given a formula F in CNF, find an assignment that satisfies a maximum number of clauses among all assignments. We will show that if $\texttt{SAT} \in \mathsf{P}$, then `MaxSAT` can be solved exactly by a polynomial time deterministic Turing machine.

a) Let F be a formula with n variables. We can view the elements in $\{0,1\}^n$ as possible assignments. Let $\leq$ be the lexicographic ordering on $\{0,1\}^n$.
Show that the following language is in Σ_2^P:

$$\begin{aligned} A = \{\langle F, u, t\rangle \mid\ & F \text{ is a formula in CNF and } u \text{ and } t \text{ are assignments} \\ & \text{such that there exists an assignment } a \text{ with } u \leq a \leq t \\ & \text{that satisfies as least as many clauses as any other assignment } b\}. \end{aligned}$$

b) Show that there is a deterministic polynomial time oracle Turing machine M with oracle A (as above) that solves `MaxSAT`.

c) Show that if $\texttt{SAT} \in \mathsf{P}$, then $\mathsf{PH} = \mathsf{P}$.

d) Conclude that if `SAT` is in P, then `MaxSAT` can be solved by a polynomial time deterministic Turing machine (without oracle).