Assignment 6, Complexity Theory, WS 13/14

Markus Bläser, Thatchaphol Saranurak
http://www-cc.cs.uni-saarland.de/course/42/

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Exercise 6.1 Consider the following game, generalized geography, played on a directed graph $G = (V, E)$ by two players: We start in a given node a. The players alternate choosing a new node where each chosen node has to be a direct successor of the current node. The players are not allowed to choose a node that has been previously chosen. The first player who cannot pick a new node looses the game.

We say that the first player has a winning strategy if he can win the game no matter which moves the second player plays. Let

$$GG = \{(G, a) \mid \text{the first player has a winning strategy on } G\}.$$

Prove that GG is PSPACE-complete.

Exercise 6.2 *(Bonus exercise)* Can you find a (reasonable) game such that the problem whether the first player has a winning strategy with at most $f(k)$ moves (for some fixed function f) is complete for Σ_k^P?

Exercise 6.3 An alternating Turing machine is a nondeterministic Turing machine. The states of this Turing machine are divided into existential and universal states. We assume that every computation path is finite. Consider the computation tree of such a machine on some input x. The value of a node in such a tree is inductively defined as follows: Leaves that correspond to an accepting configuration have value true, all other leaves have value false. The value of an internal existential node is the OR of the values of its children. The value of an internal universal node is the AND of the values of its children. An alternating Turing machine accepts a word x if the value of the root of the computation tree on x is true. The running time on x is the height of the computation tree.

Prove that $\mathsf{AP} = \mathsf{PSPACE}$, where $\mathsf{AP} = \bigcup_i \text{ATime}(n^i)$.