**Assignment 4, Complexity Theory, WS 13/14**

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Due: Nov. 20 2013

**Exercise 4.1** A permutation in $S_5$ is called a 5-cycle, if its graph consists of only one cycle.

a) Prove that if $\sigma$ is a 5-cycle, so is $\sigma^{-1}$.

b) Prove that there are two 5-cycles $\sigma$ and $\tau$ such that $\sigma\tau\sigma^{-1}\tau^{-1}$ is a 5-cycle.

**Exercise 4.2** Let $\sigma \in S_5$, $\sigma \neq \text{id}$. We say that a permutation program $B$ of width 5 $\sigma$-decides a language $L \subseteq \{0,1\}^n$, if for all $x \in L$, $B(x) = \sigma$, and for all $x \notin L$, $B(x) = \text{id}$. Let $\tau$ and $\sigma$ be 5-cycles. Prove the following: If $B$ is a permutation program of width 5 and length $\ell$ that $\sigma$-decides some language $L$, then there is a permutation program of the same length that $\tau$-decides $L$.

**Exercise 4.3** Let $C$ be a Boolean circuit of depth $d$ deciding a language $L \subseteq \{0,1\}^n$. Prove that there is a permutation program $B$ of width 5 and length $4^d$ that $\sigma$-decides $L$ for some 5-cycle $\sigma$.

**Exercise 4.4** Let $L$ be some language. Prove Barrington's theorem: $L \in \mathsf{NC}_1$ if and only if there is a family of bounded width permutation programs of polynomial length that decides $L$.