Assignment 3, Complexity Theory, WS 13/14

Markus Bläser, Thatchaphol Saranurak
http://www-cc.cs.uni-saarland.de/course/42/

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This is the third sheet by coincidence.

Exercise 3.1 In this exercise, we fill in the missing details of the proof of Theorem 5.7.

a) Show the following: There is an $O(\log n)$ depth and $O(n^3)$ size bounded logarithmic space uniform family of circuits C such that C_n computes the Boolean product of two given Boolean $n \times n$ matrices.

b) Show the following: There is an $O(\log^2 n)$ depth and $O(n^3 \log n)$ size bounded logarithmic space uniform family of circuits D such that D_n computes the nth Boolean power of a given Boolean $n \times n$ matrix.

Exercise 3.2 a) Prove the following lemma: For every binary tree with $n \geq 3$ nodes, there is an edge e such that removing this edge splits the tree into two trees, each having at least $n/3$ nodes.

b) Use this to prove the following: If F is a Boolean formula with n gates, then there is a depth $O(\log n)$ Boolean formula F' with $\text{poly}(n)$ gates that computes the same function as F.

Exercise 3.3 Let $W = \{1, \dots, w\}$ for some integer w. Let S_w be the symmetric group. A length ℓ permutation program B of width w with n inputs is a sequence of triples (j_i, σ_i, τ_i) with $1 \leq i \leq \ell$ and $\sigma_i, \tau_i \in S_w$. For $x \in \{0,1\}^n$, $B(x)$ is defined as follows: We set

$$\pi_i = \begin{cases} \sigma_i & \text{if } x_{j_i} = 1 \\ \tau_i & \text{otherwise} \end{cases}$$

and $B(x) = \pi_\ell \circ \cdots \circ \pi_1$. A sequence of branching programs (B_n) decides a language $L \subseteq \{0,1\}^*$ if each B_n has n inputs and for every n, there is a set $F_n \subseteq S_w$ such that for all $x \in \{0,1\}^n$, $x \in L$ iff $B_n(x) \in F_n$.

Show that there is a family of bounded width permutation programs of polynomial length that decides the language $\texttt{PARITY} = \{x \in \{0,1\}^* \mid x \text{ has an odd number of 1s}\}$.