Assignment 2, Complexity Theory, WS 13/14

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Exercise 2.1 (a.k.a. 3.1) is moved to the next sheet (which will have the number 3 by tradition).

Exercise 2.1 (a) Show that for any Boolean circuit of depth d, there is an equivalent Boolean formula of depth $O(d)$ and size $2^{O(d)}$.

(b) Prove that for any Boolean circuit C of size s, there is an equivalent circuit C' of size at most $2s + n$ such that all negation gates have depth 1 in C'.

Exercise 2.2 In this exercise, we construct a space constructible function $f(n)$ with $f \in O(\log \log n)$ and $\limsup f(n) = \infty$. Let $\ell(n)$ be the smallest integer not dividing n.

a) Prove that $\ell(n)$ is a prime power for all n.

b) Prove that $\ell(n) = O(\log n)$.

c) Prove that $f(n) = \lceil \log_2(\ell(n) + 1) \rceil$ is space constructible.

Exercise 2.3 a) Let $s(n) = o(\log n)$ be space constructible. Prove that there is a value k such that $s(n) = k$ for infinitely many n.
(Hint: Have a closer look at the proof of Lemma 1.1.)

b) Conclude that $\log \log n$ is not space constructible.

Exercise 2.4 Prove that $\mathsf{DSpace}(O(n)) \neq \mathsf{P}$.