Complexity Theory, WS 13/14: Solution Hints 1.

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Exercise 1.1 On input x, we run a log space bounded DTM M for computing g. Whenever this machine wants to read a bit of $f(x)$, we interrupt the computation of M. We store the position that M wants to read. We run a machine N for f on input x. This machine would write $f(x)$ on the output tape. However, we cannot afford to store $f(x)$ on the worktape. Therefore, we throw away all the bits of $f(x)$ until we produce the bit that M wants to read. (Use a counter for this.) Then we resume the computation of M. We need to store the content of the worktape of two log space bounded DTMs and two counters.

Exercise 1.2

`StrConn` $\in$ NL*:* We guess non deterministically a pair of nodes (x, y) and check if these are not connected (which we can do, since NL = co-NL). This solves $\overline{\texttt{StrConn}}$. NL = co-NL gives the rest. (Guessing a path for every pair of nodes does also work.)

CONN $\leq_{\log}$ `StrConn`*:* For every node $v \neq s, t$ we add edges (v, s) and (t, v).

If $s \rightsquigarrow t$ then G' is strongly connected. For every two nodes (v_1, v_2) we have the path $v \to s \rightsquigarrow t \to v_2$.

If the graph G' is strongly connected then $s \rightsquigarrow t$. We try to find a path $s \rightsquigarrow t$ in G' even if a path $s \rightsquigarrow t$ does not exist in G. This new path has to uses the new edges. But every new edge either starts from t or goes to s. In both cases we have a cycle. As every path from s to t can be made simple, we get a contradiction.

Exercise 1.3 We know NL = co-NL. We can use basic translation to proof our theorem. Assume we have given $M \in \mathsf{NSpace}(s)$. We then blowup the output to get it into NL. Let $L = L(M)$. We construct $L' = \{x1^{2^s - |x|} \mid x \in L\}$. This language is in NL. We can just check the input for correctness in $\log(2^s) = s$ space and run M on this input which takes s space. This language is also in co-NL with a machine M_{co}. We now construct a co-NSpace(s) time turing machine as follows. Our machine takes the input and adds "virtually" $1^{2^s - |x|}$ to our input and runs M_{co} on it. Our machine takes s space for simulating the virtual padding and $O(s)$ space for simulating M_{co}. It follows:

$$x \in L_{\text{fin}} \iff x1^{2^s - |x|} \in L(M_{\text{co}}) \iff x1^{2^s - |x|} \in L' \iff x \in L.$$

The other direction works similar.