Errata to
Introduction to Algorithms and Data Structures

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Some general ideas

- *use a different scheme for numbering (same counter for all theorem environments, and possibly even equations) – less confusion whether Lemma x, Equation x or Whatever x is meant.*
- *formulate Master theorem to yield $\Theta(\ldots)$ instead of $\mathrm{O}(\ldots)$ and/or relax notion of base cases (e.g. strict equality to inequality, $\mathbb{N} \to \mathbb{R}_{>0}$ instead of $\mathbb{N} \to \mathbb{N}$)?*
- *replace* log *by* ILog *(integer* log*) where $\lfloor log_2(x+1) \rfloor$ is meant.*
- *replace "i-th largest element" by "element of i-th order", "element of i-th rank" or the like (in chapter 4, median-of-medians-based select algorithm).*

Section 1.3, Definition 1.2 (p. 5) *reported on 2011-11-30*

3. $\Theta(f) = \mathrm{O}(f) \cap \Omega(f)$ ~~$\Omega(g)$~~,

Section 1.4.2, Lemma 1.10 (p. 8) Let $g_1, \ldots, g_\ell : \mathbb{N} \to \mathbb{N}$ ~~$\mathbb{R}_{\geq 0}$~~ be functions such that ... *reported on 2012-01-11*

Section 3.1.1 (p. 17) The last layer might be shorter and is stored in $A[2^h$~~-1~~$..$*heap-size*$]$. Here $h = \log($*heap-size*$) - 1 = \lfloor \log_2($*heap-size*$) \rfloor$ is the height of the tree, [...] *reported on 2012-01-25*

Section 3.1.3 (p. 19) Now assume we have an array $A[1..n]$ and we want to convert it into a heap. We can use the procedure Heapify in a bottom-up manner. Because the indices ~~$\{\lfloor n/2 \rfloor, \ldots, n\}$ are all leaves, the~~ $\{\lfloor n/2+1 \rfloor, \ldots, n\}$ all represent leaves, each subtree with a root ~~$j \geq \lfloor n/2 \rfloor$~~ at $j > \lfloor n/2 \rfloor$ is a heap. Then we apply Heapify and ensure the heap property in a layer by layer fashion. The correctness of the approach can be easily proven by reverse induction on i. *reported on 2011-11-18*

Section 4.2, Proof of Lemma 4.4 (p. 28) Since m is the median of the medians, $\lceil \frac{1}{2}(r-1) \rceil$ medians are larger and $\lfloor \frac{1}{2}(r-1) \rfloor$~~$-1$~~ medians are smaller than m. *reported on 2011-11-23*

reported on 2011-11-23

Section 4.2, before Remark 4.6 (p. 29) We can use Lemma 4.5 to solve (4.1). We can bound $\frac{7}{10}n+2$ from above by $\frac{11}{15}n$ for $n > 60$. Since $\frac{1}{5}+\frac{11}{15}+\frac{2}{60} = \frac{29}{30} < 1$, we get that $t(n) \leq c \cdot n$ with ~~$c = 132$~~ $c = 102$.

The parameter choices corresponding to equation (4.1) are

$$\ell = 2, \quad \epsilon_1 = \tfrac{1}{5}, \quad \epsilon_2 = \tfrac{11}{15}, \quad d = \tfrac{17}{5}, \quad N = 60, \quad e = 8.$$

Thus, $c = \max\{\frac{d}{1-(\epsilon_1+\epsilon_2+\frac{\ell}{N})}, e\} = \max\{\frac{17/5}{1-29/30}, 8\} = 102$.

reported on 2011-12-12

Section 6.1 (p. 38) If $\mathrm{Key}(r) = k$ ~~v~~, then we are done.

reported on 2011-12-12 & 2011-12-13

Section 6.1, Algorithm 26 (p. 38)

Algorithm 26 BST-search

Input: node x, key k
Output: a node $y \in T(x)$ with $\mathrm{Key}(y) = k$ if such a y exists, NULL otherwise

1: **if** $x = \mathrm{NULL}$ or $k = \mathrm{Key}(x)$ ~~$k = \mathrm{Key}[x]$~~ **then**
2: return x
3: **if** $k < \mathrm{Key}(x)$ ~~$k < \mathrm{Key}(y)$~~ **then**
4: return BST-search(Left(x), k)
5: **else**
6: return BST-search(Right(x), k)

reported on 2011-12-14

Section 7.1, Proof of Lemma 7.2 (p. 45) We show by induction on ~~n~~ h that...

reported on 2011-12-14

Section 7.2.2, before Observation 7.4 (p. 46–47) ...a virtual leaf is replaced by an internal ~~a virtual~~ node...

reported on 2011-12-14

Section 7.2.2, first table (p. 48)

	before insertion	after insertion	after rotation
Bal(x)	-1	-2	~~-1~~ 0
Bal(y)	0	-1	0
Height(T_1)	h	h	h
Height(T_2)	~~$h+1$~~ h	~~$h+1$~~ h	~~$h+1$~~ h
Height(T_3)	~~$h+1$~~ h	~~$h+2$~~ $h+1$	~~$h+2$~~ $h+1$
Height($T(x)$)	~~$h+3$~~ $h+2$	~~$h+4$~~ $h+3$	~~$h+2$~~ $h+1$
Height($T(y)$)	~~$h+2$~~ $h+1$	~~$h+3$~~ $h+2$	~~$h+3$~~ $h+2$

All numbers in rows 4–7 were decreased by exactly one.

Section 9.1 (p. 61) Of course, in the worst case, every bit has to be changed to 0 ~~is set to 1~~ and we have to flip all ~~n~~ ℓ bits (and get an overflow error). *reported on 2011-12-01 & 2011-12-03*

Section 9.1.1 (p. 62) Therefore, the total time is *reported on 2011-12-06*

$$t(n) = \sum_{i=0}^{\text{~~}n\text{~~}\,\ell-1} \lfloor \frac{n}{2^i} \rfloor \leq n \cdot \sum_{i=0}^{\text{~~}n\text{~~}\,\ell-1} \text{~~}\frac{1}{2^i}\text{~~} \leq n \sum_{i=0}^{\infty} \frac{1}{2^i} = 2n$$

and the amortized costs are [...]

Chapter 10, Theorem 10.1 (p. 71) *reported on 2011-11-23*

3. If $f(n) = \Omega(n^{\log_b a+\epsilon})$ for some $\epsilon > 0$ and ~~$a\, f(\lceil n/b \rceil) \leq d f(n)$~~ $af(\lceil n/b \rceil) \leq df(n)$ for some constant $d < 1$ and all sufficiently large n, then $t(n) = \mathrm{O}(f(n))$.

Chapter 10, Exercise 10.1 (p. 71) Let $f : \mathbb{N} \to \mathbb{N}$, $f \not\equiv 0$. Show that if f fulfills ~~$f(\lceil n/b \rceil) \leq d f(n)$~~ $af(\lceil n/b \rceil) \leq df(n)$ for some constant $d < 1$ and all sufficiently large n, then $f(n) = \Omega(n^{\log_b a+\epsilon})$ for some $\epsilon > 0$. *reported on 2011-11-23*

Chapter 10, Proof of Theorem 10.1 (p. 72) We start with the first two cases. Let $e := \log_b a$ and $\gamma := a/b^e$. ~~, respectively.~~ *reported on 2011-11-23*

Section 11.1, before Exercise 11.1 (p. 74) The chromatic number $\chi(G)$ of a graph G is the smallest number k such that there is a proper k-coloring of G. *reported on 2012-01-29*

Section 11.1, after Exercise 11.1 (p. 74) [...] how can we decide whether G has a proper k-coloring? First, we can try all ~~proper~~ k-colorings. *reported on 2012-01-29*

Chapter 12, after Exercise 12.1 (p. 80) A *cycle* is a walk such that $v_0 = v_k$, $k > 0$ (if G is directed) or ~~$k > 1$~~ $k > 2$ (if G is undirected), ... *reported on 2012-01-11*

Section 12.1 (p. 81 bottom) With an adjacency-list~~matrix~~-representation, however, ... *reported on 2012-01-11*

Section 12.2.2 (p. 85) If we have an adjacency-matrix~~list~~ representation, then the running time is $\mathrm{O}(|V|^2)$. *reported on 2012-01-11*

reported on 2012-01-19

Section 13.2, Proof of Theorem 13.2 (p. 90) [...] It remains to prove why this spanning tree is in fact minimal. Assume that e is not of minimal weight, i.e. there exists an edge f with lower weight. Thus, f would have been handled by the algorithm before e (line 5). Since S is a connected component of E_T it holds that $E_T \cup \{e\}$ used to be acyclic for all previous iterations of the algorithm. But then, f would have already been added to E_T, contradicting the fact that f is an edge of the cut of S.

Hence, no f with lower weight exists, so e is an edge of minimal weight in the cut $(S, V \setminus S)$, and by Theorem 13.1, the spanning tree augmented by e is minimal.

reported on 2012-02-12

Section 14.1, Algorithm 52 (p. 94)

Algorithm 52 Relax

Input: nodes u and v with $(u, v) \in E$
Output: $d[v]$ and $p[v]$ are updated
  if $d[v] > d[u] + w((u, v))$ **then**
    $d[v] := d[u] + w((u, v))$
    $p[v] := u$

reported on 2012-02-12

Section 14.2, after Algorithm 53 (p. 95) If we implement Q by an ordinary array, then the Insert and ~~Decrease-min~~ Decrease-key operations ~~that~~ take time O(1) while Extract-min takes O($|V|$). [...] If we implement Q with ~~binary~~ binomial or binary heaps, then ...