Saarland
Informatics Campus

Assignment 12, Approximation Algorithms
Summer term 2017

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`http://www-cc.cs.uni-saarland.de/course/61/`

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Exercise 12.1 (10 Points) (Exercise 4.2 from [1])
Let $\tau = S \subseteq V : x(E[S]) = |S| - 1\}$ where x is an extreme point solution to the spanning tree polytope. Solving the linear program using the ellipsoid method enables us to get a set family $\mathcal{F} \subseteq \tau$ such that constraints for sets in $\mathcal{F}$ are linearly independent and span the tight constraints in τ. But $\mathcal{F}$ need not be laminar. Give a polynomial time algorithm that, given $\mathcal{F}$, returns a laminar family $\mathcal{L} \subseteq \tau$ such that constraints for sets in $\mathcal{L}$ are independent and span all the tight constraints in τ.

Exercise 12.2 (10 Points) (Exercise 4.4 from [1])
Argue that in the iterative relaxation algorithm for bounded degree spanning trees from Lecture 12, one only needs to compute an optimal extreme point solution once initially, after that one can modify the current solution to obtain an extreme point solution for the next iteration in a simple way. (Observe that the new solution does not have to be optimal. It has to be an extreme point solution, and the cost should not increase.)

Exercise 12.3 (10 Points) Suppose, instead of violating the degrees by one, we allow to violate the degrees by two. We use the algorithm from Lecture 12, but replace $B_v + 1$ by $B_v + 2$.

We want to find another token passing argument. Instead of one token, each edge obtains two tokens. But now, we only want to pass entire tokens, not fractional ones. The aim is to assign two tokens to each set in $\mathcal{L}$ and two tokens to each set in T. Find a local token passing argument that achieves this. Hint: you may assign extra tokens to sets from $\mathcal{L}$ and re-assign them to super-sets.

Exercise 12.4 (10 Points) (Inspired by Exercise 4.5 from [1])
In an instance of the minimum bounded weighted-degree spanning tree we are given a graph $G = (V, E)$ and a cost function $c\colon E \to \mathbb{R}^+$, a weight function $w\colon E \to \mathbb{R}^+$, and a degree bound B_v for each vertex v. The goal is to find a minimum cost tree such that $\sum_{e \in \delta(v)} w(e) \leq B_v$ for all $v \in V$. An (a, b)-approximation algorithm for this problem is an algorithm such that the cost of the tree is at most a times the minimal cost, and the degree constraints are violated by at most a factor b. Find an (a, b) approximation algorithm for weighted-degree spanning tree with constant a and b.

References

[1] Lap-Chi Lau, R. Ravi, and Mohit Singh. *Iterative Methods in Combinatorial Optimization.* Cambridge University Press, New York, NY, USA, 1st edition, 2011.