Saarland
Informatics Campus

Assignment 11, Approximation Algorithms
Summer term 2017

Tobias Mömke, Hang Zhou
http://www-cc.cs.uni-saarland.de/course/61/

Due: 11 July 2017

Exercise 11.1 (10 Points) Let A be the constraint matrix of a linear program with constraints $Ax \geq b$. Suppose that for all i, $x_i > 0$. Show that if A has linearly dependent columns, x cannot be an extreme point solution.

Exercise 11.2 (10 Points)
(a) Show that every laminar family composed of subsets of a set V contains at most $2|V| - 1$.
(b) Show that there are laminar families with $2|V| - 1$ sets.

Exercise 11.3 (10 Points) (Exercise 4.1 from [1] in the script of Lecture 11.)

Consider the following partition LP for the minimum spanning tree problem. Let $\pi = \{V_1, V_2, \ldots, V_\ell\}$ be a partition of the vertex set V, and let $|\pi| = \ell$ denote the size of the partition. Define $\Delta(\pi)$ to be the set of edges with endpoints in different sets in the partition π. In any spanning tree, there are at least $|\pi| - 1$ edges in $\Delta(\pi)$ for a partition π of V. Show that the partition LP is equivalent to (T).

$$\text{minimize} \quad \sum_{e \in E} c_e x_e \tag{1}$$

$$\text{s.t.} \quad x(\Delta(\pi)) \geq |\pi| - 1 \quad \text{for all partitions } \pi \text{ of } V \tag{2}$$

$$x(E[V]) = |V| - 1 \tag{3}$$

$$x_e \geq 0 \quad \text{for all } e \in E \tag{4}$$

Exercise 11.4 (10 Points) Give a formal proof of Lemma 4 from Lecture 11.