Saarland Informatics Campus

Assignment 10, Approximation Algorithms
Summer term 2017

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http://www-cc.cs.uni-saarland.de/course/61/

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In the exercises, we study a new bootstrap problem under another formulation of the noise: the definition of the noise level at a white/blue node remains the same as in the lecture, while the noise level at a red node is the sum of the noise levels among all input gates that are not marked, that is:

$$\ell(v) = \begin{cases} 0 & \text{if } v \text{ is white,} \\ \max\limits_{(u,v)\in E} \ell(u) \cdot \mathbb{1}_{V\setminus S}(u) & \text{if } v \text{ is blue,} \\ \sum\limits_{(u,v)\in E} \ell(u) \cdot \mathbb{1}_{V\setminus S}(u) & \text{if } v \text{ is red.} \end{cases}$$

Exercise 10.1 (10 Points)
As a counterpart of an *interesting path* defined in the lecture, we now define an *interesting tree*, which is a connected subgraph of G with a tree structure, such that it contains exactly $L+1$ red vertices.

Similar to Fact 2.1 in the lecture, show that a set of marked vertices is a feasible solution to the new bootstrap problem if and only if every interesting tree has a non-root vertex that is marked.

Exercise 10.2 (10 Points)
Provide a linear programming formulation for the new bootstrap problem based on Exercise 10.1.

Exercise 10.3 (10 Points)
Show that for the new bootstrap problem, the hardness result still holds:

Let $L \geq 2$ be an integer parameter. For any $\epsilon > 0$, it is NP-hard to approximate the new bootstrap problem within a factor of $L - \epsilon$, assuming the Unique Games Conjecture.

Exercise 10.4 (10 Points)
Can you round the linear program in Exercise 10.2 to obtain an L-approximation for the new bootstrap problem? Either explain the difficulties in extending the rounding argument from the lecture, or if you succeed in obtaining an L-approximation algorithm for the new bootstrap problem, then congratulations: You can then publish a research paper!