Saarland
Informatics Campus

Assignment 6, Approximation Algorithms
Summer term 2017

Tobias Mömke, Hang Zhou
http://www-cc.cs.uni-saarland.de/course/61/

Due: 6 June 2017

Exercise 6.1 (20 Points)
Let $U \subseteq V$ be any subset of vertices. We want to show that there is a polynomial-time 2-approximation algorithm for minimum cost Steiner tree on U.

a) As a warm up, first, we suppose that the graph G is complete and that the edge costs obey the triangle inequality, i.e., $c_{ij} \leq c_{ik} + c_{kj}$ for all $i, j, k \in V$. Let $G[U]$ be the graph induced on U, that is, $G[U]$ contains the vertices in U and all edges from G that have both endpoints in U. Consider computing a minimum spanning tree in $G[U]$. Show that this gives a 2-approximation algorithm for the minimum-cost Steiner tree problem.

b) Now we consider the general case where the graph is not necessarily complete and the edge costs do not obey the triangle inequality. Let c'_{ij} be the cost of the shortest path from i to j in G using input edge costs c. Consider running the algorithm above in the complete graph G' on U with edge costs c' to obtain a tree T'. To compute a tree T in the original graph G, for each edge $(i, j) \in T'$, we add to T all edges on a shortest path from i to j in G. Show that this is a 2-approximation algorithm for the minimum-cost Steiner tree problem on general instances.

Exercise 6.2 (10 Points)
Show Lemma 1 of the lecture:

$$\sum_{e \in T} c_e \leq \frac{2}{\alpha} \sum_{e \in E} c_e x_e^*.$$

Exercise 6.3 (10 Points)
Derandomize the algorithm in Section 4 of the lecture to obtain a deterministic $\frac{1}{1-e^{-1/2}}$-approximation algorithm for the prize-collecting Steiner tree problem.

(Hint: there is a polynomial number of distinct values of y_i^*.)