Saarland Informatics Campus

Assignment 3, Approximation Algorithms
Summer term 2017

Tobias Mömke, Hang Zhou
http://www-cc.cs.uni-saarland.de/course/61/

Due: 16 May 2017

Exercise 3.1 (10 Points)
Show Lemma 2 of the lecture:

A $(1 + O(\epsilon))$-approximation of the scaled and perturbed instance determines a $(1 + O(\epsilon))$-approximation of the original instance.

Exercise 3.2 (10 Points)
(a) Show that the proof of Lemma 3 leads to a correct solution, i. e., if a portal is crossed twice from the same side then we can remove one of the crossings and obtain a feasible solution.

(b) Show that we can remove self-intersections of a tour τ without increasing the length.

Exercise 3.3 (10 Points)
(a) Show that an optimal TSP solution τ crosses at most $2 \cdot w(\tau)$ grid lines.

(b) Show that the analysis presented in the lecture cannot be immediately used for the three-dimensional Euclidean space.

Exercise 3.4 (10 Points)
The two dimensional Euclidean Steiner tree problem is defined as follows. We are given a set of n points in $\mathbb{R}^2$. All points in $\mathbb{R}^2$ can be used as auxiliary vertices (so called Steiner vertices). Thus there are infinitely many Steiner vertices.

We aim to find the smallest tree that contains all of the n vertices.

Example. Suppose there are vertices at the coordinates $(0, 0)$, $(2, 0)$, $(1, \sqrt{3})$ (the corners of an equilateral triangle). Then a minimum weight spanning tree has weight 4. The optimal Steiner tree uses the center of the triangle $(1, \sqrt{3}/3)$ as a Steiner vertex, which leads to a solution of weight $3 \cdot 2\sqrt{3}/3 = 2\sqrt{3} < 4$.

Show that there is a PTAS for the two dimensional Euclidean Steiner tree problem. (You can refer to the script when appropriate.)