Assignment 12, Complexity Theory, SoSe 15

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Due: never

Here are some sample exercises for the exam. The size of this assignment is about twice the size of the exam.

Exercise 12.1 Show that if $\mathsf{NSpace}(\log n) \cap \{0\}^* = \mathsf{DSpace}(\log n) \cap \{0\}^*$ then $\mathsf{NSpace}(n) = \mathsf{DSpace}(n)$.
(Hint: some sort of translation)

Exercise 12.2 Let $\mathsf{2EXP} = \bigcup_i \mathsf{DTime}(2^{2^{O(n^i)}})$. Prove that

$$\mathsf{EXP}^{\mathsf{EXP}} = \mathsf{2EXP}.$$

Exercise 12.3 Which of the following statements are true, which are false? Give a short proof for each of your answers! "Most likely not" might also be an answer. In this case, you should derive some unplausible inclusion of complexity classes from it.

a) $\{x \in \{0,1\}^* \mid x = x^{\mathrm{rev}}\} \in \mathsf{DSpace}(\log\log\log n)$.

b) For every complexity class C and $A \in \mathsf{C}$: If A is C-complete under polynomial time many one reductions, then $\mathsf{C} \subseteq \mathsf{P}^A$.

c) If $\mathsf{NC}_2 \subseteq \mathsf{L}$, then $\mathsf{DSpace}(s(n)) = \mathsf{NSpace}(s(n))$ for all space constructible $s \geq \log n$.

d) For all languages L, L': If L is NP-complete and L' is $\mathsf{co\text{-}NP}$-complete, then $L \cap L'$ is $\mathsf{NP} \cap \mathsf{co\text{-}NP}$-complete.

e) If $\mathsf{PH} = \mathsf{PSPACE}$, then PH collapses.

f) For all NP-relations R: If $\#R$ is $\#\mathsf{P}$-complete under polynomial time Turing reductions, then $L(R)$ is NP-complete under polynomial time Turing reductions.

g) If $\mathsf{P} = \mathsf{PSPACE}$, then $\mathsf{RP} = \mathsf{BPP}$.

h) The permanent is $\#\mathsf{P}$-hard under parsimonious reductions.

Exercise 12.4 a) Assign to each node v of $G = (V, E)$ a weight $w_v \in \{1, \dots, 2n\}$ uniformly at random. For a subset $U \subseteq V$, the weight of U is $w(U) = \sum_{u \in U} w_u$. Let $1 \le k \le |V|$. Show that with probability $\ge 1/2$, there is exactly one k-clique of *maximum* weight, if G has at least one k-clique.

b) Let $n = |V|$ and $1 \le k \le n$. Next replace each node $v \in V$ by a $(2nk + w_v)$-clique. For every edge $\{u, v\} \in E$, connect every node of the clique belonging to u with every node of the clique belonging to v. Let $G' = (V', E')$ be the resulting graph. Show that with probability $\ge 1/2$, there is an $1 \le r \le 2nk$ such that

$$G \text{ has no } k\text{-clique} \Longrightarrow G' \text{ has no } (2nk^2 + r)\text{-clique.}$$

$$G \text{ has a } k\text{-clique but no } (k+1)\text{-clique} \Longrightarrow G' \text{ has } \textit{exactly} \text{ one } (2nk^2 + r)\text{-clique.}$$

c) Show that if there is a polynomial time bounded deterministic Turing machine that on input (H, k) outputs a k-clique of H if H has exactly one k-clique (and otherwise can do what it wants), then $\mathsf{NP} = \mathsf{RP}$.

d) We know that $\#\texttt{SAT}$ is $\#\mathsf{P}$-complete under parsimonious reductions. Use this fact to deduce the Valiant–Vazirani theorem from c).

Exercise 12.5 Recall that a language L is in the class IP, if there is a prover-verifier pair (P, V) such that for all x:

$$x \in L \implies \Pr[(P, V)(x) = 1] \ge 2/3 \tag{1}$$

$$x \notin L \implies \Pr[(\hat{P}, V)(x) = 1] \le 1/3 \text{ for all provers } \hat{P} \tag{2}$$

We now change the error probabilities and keep everything else as it is.

a) Show that if we replace the probability $1/3$ by 0 in equation (2), then such interactive proofs will characterize the class NP.
(Hint: Guess the random string and the conversation.)

b) Show that if we replace the probability $2/3$ by 1 in equation (1), then we will still get IP.
(Hint: The protocol for QBF.)