Assignment 10, Complexity Theory, SoSe 15

Markus Bläser, Holger Dell, Karteek Sreenivasaiah
http://www-cc.cs.uni-saarland.de/course/47/

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Exercise 10.1 Show that we can deterministically decide in polynomial time whether the permanent of an integer matrix is odd.

Exercise 10.2 For an undirected multigraph G, let $M(G;x)$ be the *matching polynomial* of G, that is, the polynomial

$$M(G;x) = \sum_{\text{matching } M \subseteq E(G)} x^{|M|}\,.$$

Recall that $M \subseteq E(G)$ is a *matching* if, for all $e, e' \in M$ with $e \neq e'$, we have $e \cap e' = \emptyset$.

a) Prove that the function $G \mapsto M(G;0)$ can be computed in polynomial time.

b) Let $m_k(G) \in \mathbb{N}$ be such that $M(G;x) = \sum_{k=0}^{n} m_k(G)x^k$. Prove that $G \mapsto m_n(G)$ is $\#\mathsf{P}$-hard under polynomial-time many-one reductions.

c) For all fixed $a \in \mathbb{Q} \setminus \{0\}$, prove that $G \mapsto M(G;a)$ is $\#\mathsf{P}$-hard to compute under polynomial-time Turing reductions.
Hint: First construct from G a graph G_t such that $M(G;tx) = M(G_t;x)$ for $t \in \mathbb{N}$. Then use polynomial interpolation to compute the coefficients from sufficiently many evaluation points.

Exercise 10.3 Let R be an NP relation. Let $\oplus R$ be the language

$$\{x \mid \text{the number of } y \text{ with } R(x,y) = 1 \text{ is odd}\}\,.$$

Prove that if $\#R$ is $\#\mathsf{P}$-hard under parsimonious reductions, then $L(R)$ is NP-hard under many-one reductions and $\oplus R$ is $\oplus P$-hard under many-one reductions.