Assignment 6, Complexity Theory, SoSe 15

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Exercise 6.1 Show that $\mathtt{QBF}\Sigma_k$ is Σ_k^P-complete under polynomial-time many-one reductions.

Exercise 6.2 If PH has complete problems (under polynomial-time many-one reductions), then PH collapses.

Exercise 6.3 Let $A \subseteq \Sigma^*$. Prove that $(\mathsf{co\text{-}NP})^A = \mathsf{co\text{-}}(\mathsf{NP}^A)$.

Exercise 6.4 Let $\mathtt{CMP}$ (circuit minimization problem) be the language of the encodings of all minimal (with respect to size) Boolean circuits.

a) Show that $\mathtt{CMP} \in \Pi_2^{\mathsf{P}}$.

b) Show that if $\mathtt{SAT} \in \mathsf{P}$, then $\mathtt{CMP} \in \mathsf{P}$.

This is a somewhat paradox situation. In the second case, we are given an algorithm for $\mathtt{SAT}$ and this places $\mathtt{CMP}$ into P. In the first case, we are essentially given an oracle for $\mathtt{SAT}$ ($\Pi_2^{\mathsf{P}} = \mathsf{co\text{-}NP}^{\mathsf{NP}}$). This oracle solves the same problem as the algorithm above, but it is not clear whether this is enough to solve $\mathtt{CMP}$ in polynomial time (most likely not).

Exercise 6.5 Prove that $\exists\mathsf{L} = \mathsf{NP}$.