Assignment 5, Complexity Theory, SoSe 15

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Exercise 5.1 The *circuit value problem* CVAL is the following problem: Given (the encoding of) a circuit C with n input gates and one output gate, and a string $x \in \{0,1\}^n$, the goal is to decide whether $C(x) = 1$. Prove that CVAL is P-complete under logarithmic-space many-one reductions.

Hint: We already proved that $\texttt{CVAL} \in \mathsf{P}$. To show that CVAL is P-hard, first prove Remark 4.6 in the lecture notes.

Exercise 5.2 Let $f : \{0,1\}^n \to \{0,1\}$ be a Boolean function. Show that the mutlilinear polynomial $p \in \mathbb{C}[x_1, x_2, \ldots, x_n]$ that represents f exactly is indeed unique. That is, if p and p' are two polynomials with $f(x) = p(x) = p'(x)$ for all $x \in \{0,1\}^n$, then p and p' are identical as polynomials over $\mathbb{C}[x_1, \ldots, x_n]$.

Exercise 5.3 Show that, if A is downward self-reducible, then $A \in \mathsf{PSPACE}$.

Exercise 5.4 Let M be a deterministic Turing machine that only queries oracle strings that are shorter than the input string. Show that, if $A = L(M^A)$ and $B = L(M^B)$, then $A = B$.

Hint: prove by induction over n that $A^{\leq n} = B^{\leq n}$ holds.

Exercise 5.5 If co-NP contains an NP-complete problem, then $\mathsf{NP} = \text{co-}\mathsf{NP}$.

Exercise 5.6 Prove the following:

a) $\texttt{FACTOR} \in \mathsf{NP}$.

b) $\texttt{FACTOR} \in \text{co-}\mathsf{NP}$. (You can use the result by Agrawal, Kayal, and Saxena that $\texttt{PRIMES} \in \mathsf{P}$; that is, we can determine in polynomial time whether a given positive integer x is a prime number or not, where the integer x is encoded in binary)