



Assignment 4, Complexity Theory, SoSe 15

Markus Bläser, Holger Dell, Kartteek Sreenivasaiah
<http://www-cc.cs.uni-saarland.de/course/47/>

Due: May 27, 2015, 11:00

Exercise 4.1 Prove that for any Boolean circuit C with n inputs and of size s , there is an equivalent one C' of size $\leq 2s + n$ such that all negations have depth 1 in C' . (Hint: De Morgan's law.)

Exercise 4.2 Construct a circuit family \mathcal{C} for $\text{Th}^2 = \{x \in \{0,1\}^* \mid x \text{ has at least two 1s}\}$ such that C_n satisfies all of the following resource constraints simultaneously:

- Depth: $O(1)$.
- Size: $O(\log n)$.
- # Edges: $O(n \log n)$.
- Fan-in: Unbounded.

(Hint: Let $x \in \text{Th}_n^2$. Then there exists $i, j \in [n]$, $i \neq j$, such that $x_i = x_j = 1$. Note that since $i \neq j$, there is some $k \in [\log n]$ such that the k^{th} bit of i and j are different. Use this fact to partition the input positions in a clever way.)

Exercise 4.3 If $L \subseteq \{0,1\}^*$ is decided by a s size bounded and t time constructible family of circuits C , then show that

$$L \in \text{DTime}(t + s \log^2 s).$$