Assignment 3, Complexity Theory, SoSe 15

Markus Bläser, Holger Dell, Karteek Sreenivasaiah
http://www-cc.cs.uni-saarland.de/course/47/

Due: May 20, 2015, 11:00

Exercise 3.1 (Translation) Complete the proof of the Immerman-Szelepcsényi Theorem by showing the following: Let s_1, s_2, and f be space constructible such that for all n, $s_1(n) \geq \log n$, $s_2(n) \geq \log n$, and $f(n) \geq n$. If

$$\mathrm{DSpace}(s_1(n)) \subseteq \mathrm{DSpace}(s_2(n)),$$

Then,

$$\mathrm{DSpace}(s_1(f(n))) \subseteq \mathrm{DSpace}(s_2(f(n))).$$

(Hint: Mimic the proof of the time translation done in class. If the space bounds are sublinear, then we cannot explicitly pad with %s. We do this virtually using a counter counting the added %s.)

Exercise 3.2 Let M be a symmetric $(n \times n)$ random walk matrix over $[0,1]$, that is, $\sum_j M_{i,j} = 1$ for all i. Let $\|x\|_p = (\sum_i |x_i|^p)^{1/p}$ be the p-norm of x. Let λ_i be the eigenvalues of M, sorted such that $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$ holds. Let $x \in \mathbb{R}^n$. Prove:

a) $\|x\|_2 \leq \|x\|_1 \leq \sqrt{n} \cdot \|x\|_2$.

b) $\|Mx\|_1 = \|x\|_1$ holds if for all i we have $x_i \geq 0$.

c) $\lambda_1 = 1$. What is the corresponding eigenvector u so that $Mu = \lambda_1 u$?

d) If $x \perp u$, then $\|Mx\|_2 \leq |\lambda_2| \cdot \|x\|_2$.

e) $\|x\|_\infty \leq \|x\|_2$ where $\|x\|_\infty := \max_i |x_i|$.

f) Bonus 1: $\|x\|_\infty = \lim_{p\to\infty} \|x\|_p$.

g) Bonus 2: Let G be a non-empty simple graph with potential self-loops. Then $\lambda_2 = \lambda_1$ holds for its normalized adjacency matrix if and only if G is not connected.

h) Bonus 3: Let G be a non-empty, connected, and simple graph with potential self-loops. Then $\lambda_2 = 0$ holds for its normalized adjacency matrix if and only if G is a clique with all self-loops.

i) Bonus 4: Let G be a non-empty, connected, and simple graph with potential self-loops. Then $\lambda_1 = -\lambda_2$ holds for its normalized adjacency matrix if and only G is bipartite.

Exercise 3.3 Show that, for every unary language, there is a circuit family of constant size deciding it.