



Assignment 2, Complexity Theory, SoSe 15

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Due: May 13, 2015, 11:00 (**This is a Wednesday!**)

Exercise 2.1 Prove that logarithmic space computable functions are closed under composition, that is, if $f, g : \{0, 1\}^* \rightarrow \{0, 1\}^*$ are both computable by a deterministic logarithmic space bounded Turing machine, so is their composition $f \circ g$.

Exercise 2.2 Prove that UCONN is hard for L (under logarithmic space reductions).

Exercise 2.3 (*bonus exercise*) A function $s : \mathbb{N} \rightarrow \mathbb{N}$ is called fully space constructible, if there is a deterministic Turing machine that on every input of length n uses *exactly* $s(n)$ cells on the work tape.

- a) Prove that $g(n) := \lceil \log_2(f(n) + 1) \rceil$ is fully space constructible, where $f(n)$ is the smallest integer not dividing n .
- b) Prove that $g(n) = O(\log \log n)$.
- c) Prove that every fully space constructible function in $o(\log n)$ has to equal some constant value infinitely often.