Assignment 1, Complexity Theory, SoSe 15

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Due: Apr 30, 2015

If not stated otherwise, every exercise is worth 4 points (regardless of its difficulty).

Throughout the whole exercise sheet, we fix a Gödel numbering of Turing machines over the alphabet $\{0, 1\}$. We will only consider input strings from $\{0, 1\}^*$.

Definition 1 *Let $x, y \in \{0, 1\}^*$.*

a) The Kolmogorov complexity $K(x|y)$ of x given y is the length of the shortest encoding of a Turing machine that on input y outputs x.

b) The Kolmogorov complexity $K(x)$ of x is defined as $K(x) := K(x|\epsilon)$.

Exercise 1.1 Let $n \in \mathbb{N}$.

a) Prove that $K((01)^n) \leq \log n + O(1)$.

b) Prove that $K((01)^n|n) \leq O(1)$. Here $K(x|n)$ means that we get the natural number n in binary as an input

Exercise 1.2 Prove that there is at least one string x of length n with $K(x|n) \geq n$. Such a string is called *Kolmogorov random*.

Exercise 1.3 Prove that the mapping $x \mapsto K(x)$ is not computable.

Exercise 1.4 Consider a 1-tape deterministic Turing machine M that accepts the language COPY.

a) Let w be a string of length n. Consider the crossing sequence of M on input $w0^n\#w0^n$ at position i for some $n + 1 \leq i \leq 2n$. Prove that if c is the length of this crossing sequence, then $K(w) \leq O(\log n) + c$.

b) Use this to reprove Theorem 1.5.