Assignment 13, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 13.1 (10 Points) For a finite set E (the alphabet), L is called a *greedoid language* if (a) the empty string ϵ is in L; (b) for any word $x_1 x_2 \cdots x_n$ in L, $x_i \neq x_j$ for $i \neq j$; (c) if $x_1 x_2 \cdots x_n \in L$ then also $x_1 x_2 \cdots x_{n-1}$; (d) if $x, y \in L$ with $|x| > |y|$ then there is a symbol x_i in x such that $y x_i \in L$.

Show that a language L over E is a greedoid language if and only if $(E, \mathcal{F})$ is a greedoid with $\mathcal{F} := \{\{x_1, x_2, \ldots, x_n\} : x_1 x_2 \cdots x_n \in L\}$.

Exercise 13.2 (10 Points) Let E be a finite set and let $f \colon 2^E \to \mathbb{R}_+$ be a submodular function such that $f(\{e\}) \leq 2$ for each $e \in E$.

Consider the following two problems. (a) Find a maximum cardinality set $X \subseteq E$ with $f(X) = 2|X|$, where f is given by an oracle access. This problem is called the *polymatroid matching problem.*

(b) For a matroid $(E, \mathcal{F})$ with $E = E_1 \cup E_2 \cup \cdots \cup E_k$ where the E_i are pairwise disjoint pairs of elements, find a maximum cardinality set $I \subseteq \{1, 2, \ldots, k\}$ with $\bigcup_{i \in I} E_i \in \mathcal{F}$. This problem is called the *matroid parity problem.*

Show that both problem (a) polynomially reduces to problem (b) and problem (b) polynomially reduces to (a).