Assignment 12, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 12.1 (10 Points) Let $G = (V, E)$ be an undirected graph. Find a polynomial time algorithm that colors the edges with a minimum number of colors such that no circuit in G is monochromatic (i. e., not all edges of a circuit have the same color).

Exercise 12.2 (10 Points) Let $G = (V, A)$ be a complete directed graph with edge weights $w : A \to \mathbb{R}_+$, $s \in V$ a special vertex, and $K \in \mathbb{N}$ a number. Show that the problem to find a minimum weight subgraph H of G with K edge-disjoint paths from s to any vertex in $V \setminus \{s\}$ reduces to the weighted matroid intersection problem.

Hint: Consider the unions of K forests.
Let $D = (V, A)$ be a digraph and $s \in V$. You may use the following results:
(i) Suppose there are k arc-disjoint paths from s to any vertex but removing any arc destroys this connectivity property. Then every vertex except s has exactly k entering arcs.
(ii) The maximum number of arc-disjoint spanning arborescences rooted at s equals the minimum number of arcs in an s-cut (i. e., the number of arcs leaving some set $S \subset V$ that contains s).