Assignment 11, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 11.1 (10 Points) Before solving this exercise, please read Section 13.3 of Schrijver, Combinatorial Optimization. (Alternatively, you may use the result of Exercise 14, Section 5, Korte, Vygen, Combinatorial Optimization.)

Let $T = (V, E)$ be a tree and let r be a special vertex (the root). Furthermore, for $i = 1, \dots, k$, let s_i and t_i be vertices such that there is a path from r via s_i to t_i in T (without repetitions). Let P_i be the path from s_i to t_i (i. e., its set of edges).

We want to find a minimum cardinality set $S \subseteq E$ of edges in T such that removing S disconnects all pairs s_i,t_i.

Consider the linear program

$$\begin{array}{ll} \min \sum_{e \in E} x_e & \\ \sum_{e \in P_i} \geq 1 & 1 \leq i \leq k \\ x_e \geq 0 & e \in E \end{array}$$

Show that the constraint matrix of the LP is totally unimodular and thus the problem can be solved in polynomial time.

Exercise 11.2 (10 Points) Let $G = (V, E)$ be an edge-weighted graph. Let M be a set of components of an induced subgraph of G in which every vertex has a degree of at most 1 (i. e., M contains subgraphs that have one or two vertices each).

For $f \in M$, let $\mathcal{P}_f$ be the set of paths P in G such that its internal vertices (all except the two end-vertices) are exactly those from f. An *earmuff* (for M in G) is a set of paths $\{\mathcal{P}_f \mid f \in F\}$, where $F \subseteq M$ and $P_f \in \mathcal{P}_f$, such that $(V, \bigcup_{f \in F} E(P_f))$ is a forest.

Show that the problem to find a maximum weight earmuff can be solved by finding a maximum weight common independent set of two matroids.