Assignment 8, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 8.1 (10 Points) Let G be a graph. Let $\mathcal{I}$ be the family of sets $X \subseteq V(G)$ for that a maximum matching exists that covers no vertex in X. Prove that $(V(G), \mathcal{I})$ is a matroid.

Exercise 8.2 (10 Points) A family $\mathcal{F}$ of sets is said to be *laminar* if, for any two sets $A, B \in \mathcal{F}$, we have that either

(i) $A \subseteq B$ or

(ii) $B \subseteq A$ or

(iii) $A \cap B = \emptyset$.

Suppose that we have a laminar family $\mathcal{F}$ of subsets of E and a nonnegative integer $k(A)$ for every set $A \in \mathcal{F}$. Show that $(E, \mathcal{I})$ defines a matroid (called the laminar matroid) where

$$\mathcal{I} = \{X \subseteq E \mid |X \cap A| \leq k(A) \text{ for all } A \in \mathcal{F}\}.$$

What is the rank function of the laminar matroid?