



## Assignment 7, Selected Topics in Combinatorial Optimization, Summer term 2014

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<http://www-cc.cs.uni-saarland.de/course/44/>

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Due: 4 June 2014

**Exercise 7.1 (8 Points)** Prove that every graphic matroid is representable over every field.

**Exercise 7.2 (8 Points)** Let  $\mathcal{S} = S_1, S_2, \dots, S_n$  be a family of sets. A set  $T$  is a *transversal* of  $\mathcal{S}$  if there exist distinct elements  $s_1 \in S_1, s_2 \in S_2, \dots, s_n \in S_n$  such that  $T = \{s_1, s_2, \dots, s_n\}$ . Assume that  $\mathcal{S}$  has a transversal. Show that the family of transversals of  $\mathcal{S}$  is the family of bases of a matroid.

**Exercise 7.3 (4 Points)** Let  $G$  be a directed graph. A branching in  $G$  is a subgraph of  $G$  such that its underlying undirected graph is a forest and each vertex has at most one incoming arc. Consider the independence system  $(E, \mathcal{F})$  where  $E = E(G)$  and  $\mathcal{F}$  is the family of branchings in  $G$ . Show that  $(E, \mathcal{F})$  is not a matroid.