Assignment 5, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 5.1 (10 Points) Given a graph $G = (V, E)$ and $b : V \to \mathbb{N}$. Show that

$$\begin{array}{llll} (i) & x_e \geq 0 & \text{for} & e \in E \\ (ii) & x(\delta(v)) = b(v) & \text{for} & v \in V \\ (iii) & x(\delta(U)) \geq 1 & \text{for} & U \subseteq V, b(U) \text{ odd} \end{array}$$

determines the perfect b-matching polytope.

Exercise 5.2 (10 Points) Given a graph $G = (V, E)$, an edge cover of G is a set $F \subseteq E$ of edges such that each vertex $v \in V$ has at least one incident edge from F, i. e., $\delta(v) \cap F \geq 1$.

The edge cover polytope for G is determined by

$$\begin{array}{rll} 0 \leq x_e \leq 1 & \text{for} & e \in E \\ x(E[U] \cup \delta(U)) \geq \lceil \frac{1}{2}|U| \rceil & \text{for} & U \subseteq V, |U| \text{ odd.} \end{array}$$

A b-edge cover is similar to an edge cover, but $\delta(v) \cap F \geq b(v)$ where $b : V \to \mathbb{N}$ is part of the input. Show that

$$\begin{array}{llll} (i) & x_e \geq 0 & \text{for} & e \in E \\ (ii) & x(\delta(v)) \geq b(v) & \text{for} & v \in V \\ (iii) & x(E[U] \cup \delta(U)) \geq \lceil \frac{1}{2}b(U) \rceil & \text{for} & U \subseteq V \end{array}$$

determines the b-edge cover polyhedron.