# Assignment 3, Selected Topics in Combinatorial Optimization, Summer term 2014

Tobias Mömke
http://www-cc.cs.uni-saarland.de/course/44/

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**Exercise 3.1 (10 Points)** A 2-factor of a graph $G = (V, E)$ is a set of edges $F \subseteq E$ such that in $(V, F)$, each vertex has a degree of exactly two. The fractional 2-factor polytope for $G$ is determined by

$$0 \leq x_e \leq 1 \quad \text{for all} \quad e \in E \tag{1}$$
$$x(\delta(v) = 2 \quad \text{for all} \quad v \in V. \tag{2}$$

Let $x$ be a vertex of the fractional 2-factor polytope. Show that $x_e \in \{0, 1/2, 1\}$ for each $e \in E$ (that is, $x$ is half-integral) and that the support graph $G' = (V, E')$, $E' = \{e \in E : x_e > 0\}$ has no vertices of degree 4.

**Exercise 3.2 (10 Points)** Let $T \subseteq V$ be a subset of vertices of a graph $G = (V, E)$, $|T|$ even. A $T$-cut of $G$ is a cut $\delta(U)$ such that $|U \cap T|$ is odd. Let $H = (V, F)$ be a Gomory-Hu tree of $G$ with capacities $c\colon E \to \mathbb{R}_+$. Use $H$ to obtain a polynomial time algorithm that finds a minimum capacity $T$-cut of $G$.