Assignment 2, Selected Topics in Combinatorial Optimization, Summer term 2014

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Exercise 2.1 (10 Points) Carathéodory's theorem states the following:

For any $X \subseteq \mathbb{R}^n$ and x in the convex hull of X, there exist affinely independent points $x_1, x_2, \ldots, x_k \in X$ such that x is in the convex hull of $x_1, x_2, \ldots, x_k$.

(The set X could for instance be the set of all vertices of a polytope. Note that the affine independence implies $k \leq n + 1$.)

(a) Show Carathéodory's theorem.

(b) Show the rank lemma, i. e.:

Let $P = \{x : Ax = b, x \geq 0\}$ and let x be a vertex of P with $x_i > 0$ for each i. Then the number of variables is equal to the rank of A.

Exercise 2.2 (10 Points) Use the perfect matching polytope (see Theorem 11.3 of Korte, Vygen) to show that every k-regular $(k-1)$-edge-connected graph $G = (V, E)$ with $|V|$ even has a perfect matching. Show furthermore that for each edge $e \in E$, there is a perfect matching that contains e.

Note: A graph is q-edge-connected if there are q edge-disjoint paths between any two vertices or equivalently if removing any set of at most $q - 1$ edges leaves the graph connected.

Hint: find an appropriate assignment of values to the variables.